1

- ROTATION CURVES OF SPIRALS → STARS & GAS FAR OUT ORBIT TOO FAST GIVEN THE LIGHT FROM STARS AND GAS WE SEE, HENCE THERE MUST BE MORE UNSEEN MASS HOLDING THE GALAXY TOGETHER

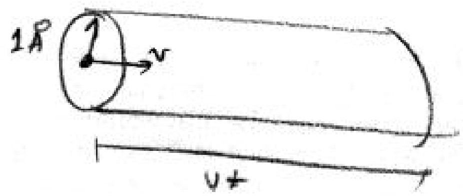
- VELOCITY DISPERSION

THE COMPONENT OF THE VELOCITY TOWARDS/AWAY FROM US (MEASURED WITH DOPPLER SHIFT) IS SO FAST THAT GIVEN ONLY THE MASS OF THE STARS & GAS WE SEE, THE CLUSTER SHOULD FLY APART. YET, THEY'RE AROUND, SO THERE MUST BE MORE UNSEEN MASS.

WE COULD USE THE VIRIAL THEOREM TO ESTIMATE $\frac{M}{L} \sim$ SEVERAL $\times 10^2 \frac{M_\odot}{L_\odot}$ IN CLUSTERS.

[2]

SIZE OF ATOM: 1 Å



$$\frac{3}{2}kT \simeq \frac{1}{2}mv^2 \qquad m = 1.7\times10^{-27}\,\mathrm{kg} \quad \text{H ATOM}$$

$$\frac{3}{2}\left(1.38\times10^{-23}\frac{\mathrm{kg\,m^2}}{\mathrm{s^2\,K}}\right)(80\,\mathrm{K}) = \frac{1}{2}\left(1.7\times10^{-27}\,\mathrm{kg}\right)v^2$$

$$v = 1.4\times10^{3}\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$n\,(\pi b^2)\,v t = 1$$

$$\left(\frac{1}{\mathrm{cm}^3}\right)\left(\frac{100\,\mathrm{cm}}{\mathrm{m}}\right)^3(\pi)\left(10^{-10}\,\mathrm{m}\right)^2\left(1.4\times10^{3}\,\frac{\mathrm{m}}{\mathrm{s}}\right)t = 1$$

$$t = 2\times10^{10}\,\mathrm{s} \doteq \boxed{700\ \text{years BETWEEN COLLISIONS}}$$

THAT'S OFTEN!

[3]

$$\nabla^2\Phi = 4\pi G\rho$$

$$= \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\Phi) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\frac{-rGM}{\sqrt{r^2+a_p^2}}\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{-GM}{\sqrt{r^2+a_p^2}} + \frac{r^2GM}{(r^2+a_p^2)^{3/2}}\right)$$

$$= \frac{1}{r}\left(\frac{rGM}{(r^2+a_p^2)^{3/2}} + \frac{2rGM}{(r^2+a_p^2)^{3/2}} - \frac{3r^3GM}{(r^2+a_p^2)^{5/2}}\right)$$

$$= \frac{GM}{r}\left(\frac{3r}{(r^2+a_p^2)^{3/2}} - \frac{3r^3}{(r^2+a_p^2)^{5/2}}\right) = \frac{GM}{r}\left(\frac{3r(r^2+a_p^2) - 3r^3}{(r^2+a_p^2)^{5/2}}\right)$$

$\Rightarrow$

3 cont'd

$$\nabla^2 \Phi = \frac{3a_p^2 \, GM}{(r^2+a_p^2)^{5/2}}$$

$$\rho = \frac{-\nabla^2\Phi}{4\pi G} = \boxed{\frac{3a_p^2}{4\pi}\frac{M}{(r^2+a_p^2)^{5/2}}}$$

$$\text{TOTAL MASS} = \int_0^\infty \rho(r) 4\pi r^2 dr = \int_0^\infty 3a_p^2 M \frac{r^2}{(r^2+a_p^2)^{5/2}} dr$$

$$= \frac{3M}{a_p}\int_0^\infty \frac{r^2/a_p^2}{(1+r^2/a_p^2)^{5/2}} dr$$

LET $r/a_p = \tan\theta$

$$dr = a_p(1+\tan^2\theta)d\theta$$

AND ALL YOU GUYS PROBABLY JUST PLUGGED MINDLESSLY INTO MATHEMATICA OR SOME SUCH...!

$$= \frac{3M}{a_p}\int_0^{\pi/2} a_p \frac{\tan^2\theta\,(1+\tan^2\theta)}{(1+\tan^2\theta)^{5/2}} d\theta \left(\frac{\cos^5\theta}{\cos^5\theta}\right)$$

$$= \frac{3M}{a_p}\int_0^{\pi/2} \frac{\sin^2\theta\cos^3\theta\,(1+\tan^2\theta)}{(\sin^2\theta+\cos^2\theta)^{5/2}} d\theta$$

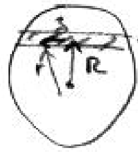
$$= 3M\int_0^{\pi/2} \sin^2\theta\cos^3\theta + \cos\theta\sin^4\theta \, d\theta$$

$$= 3M\int_0^{\pi/2} \sin^2\theta\cos\theta\,(\cos^2\theta+\sin^2\theta)d\theta$$

$$= 3M\int_0^{\pi/2} \sin^2\theta\cos\theta\, d\theta = 3M\left[\frac{\sin^3\theta}{3}\right]_0^{\pi/2}$$

$$= \boxed{M}$$ BIG SURPRISE THERE!

3 cont'd



"SURFACE DENSITY" IS MASS INTEGRATED IN THIS COLUMN

$$r^2 = R^2 + z^2$$

$$\Sigma = \int_{-\infty}^{\infty} \frac{3a_p^2}{4\pi} \frac{M}{(R^2+z^2+a_p^2)^{5/2}}\,dz$$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^{5/2}} \int_{-\infty}^{\infty} \frac{1}{\left(1+\frac{z^2}{R^2+a_p^2}\right)^{5/2}}\,dz$$

LET $\frac{z}{\sqrt{R^2+a_p^2}} = \tan\theta$

$dz = \sqrt{R^2+a_p^2}\,(1+\tan^2\theta)\,d\theta$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^2} \int_{-\pi/2}^{\pi/2} \frac{1+\tan^2\theta}{(1+\tan^2\theta)^{5/2}}\,d\theta$$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^2} \int_{-\pi/2}^{\pi/2} \frac{1}{(1+\tan^2\theta)^{3/2}} = \frac{3Ma_p^2}{4\pi(R^2+a_p^2)} \int_{-\pi/2}^{\pi/2} \frac{\cos^3\theta}{(\sin^2\theta+\cos^2\theta)^{3/2}}\,d\theta$$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^2} \int_{-\pi/2}^{\pi/2} \cos\theta\,(1-\sin^2\theta)\,d\theta$$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^2} \left[\sin\theta - \frac{\sin^3\theta}{3}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{3Ma_p^2}{4\pi(R^2+a_p^2)^2} \underbrace{\left[1-\frac{1}{3}+1-\frac{1}{3}\right]}_{4/3}$$

YAY, MORE TRIGONOMETRIC SUBSTITUTIONS!

$$\boxed{\Sigma(R) = \frac{Ma_p^2}{\pi(R^2+a_p^2)^2}}$$

~~HALF LIGHT~~: $\frac{1}{2}\frac{Ma_p^2}{\pi(a_p^2)^2} = \frac{Ma_p^2}{\pi(R^2+a_p^2)^2}$

$R^2+a_p^2 = \sqrt{2}\,a_p^2$

$R = \sqrt{\sqrt{2}-1}\,a_p = \boxed{0.644\,a_p}$

4

(a) EQ 2.11 $V_r = R_o \sin l \left[\frac{V}{R} - \frac{V_o}{R_o} \right]$ $V \approx V \approx 220 \frac{km}{s}$

$$\frac{V_r}{R_o \sin l} + \frac{V_o}{R_o} = \frac{V}{R}$$

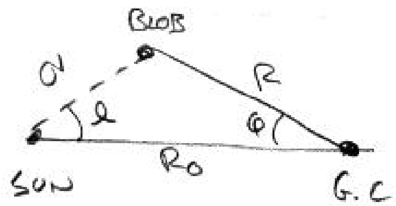
$$\frac{V_r + V_o \sin l}{R_o \sin l} = \frac{V}{R}$$

$$\boxed{R = \frac{V R_o \sin l}{V_r + V_o \sin l}}$$

$R_o = 8$ kpc

$V_o = V = 220 \frac{km}{s}$

(b)



NOTE: $\frac{\sin \varphi}{d} = \frac{\sin l}{R}$

LAW OF COSINES $R_o^2 + d^2 - 2R_o d \cos l = R^2$

$$d^2 - (2R_o \cos l)d + (R_o^2 - R^2) = 0$$

QUADRATIC FORMULA

$$d = \frac{1}{2}\left[2R_o \cos l \pm \sqrt{4R_o^2 \cos^2 l - 4R_o^2 + 4R^2}\right]$$

$$d = R_o \cos l \pm \sqrt{R_o^2(\cos^2 l - 1) + R^2}$$

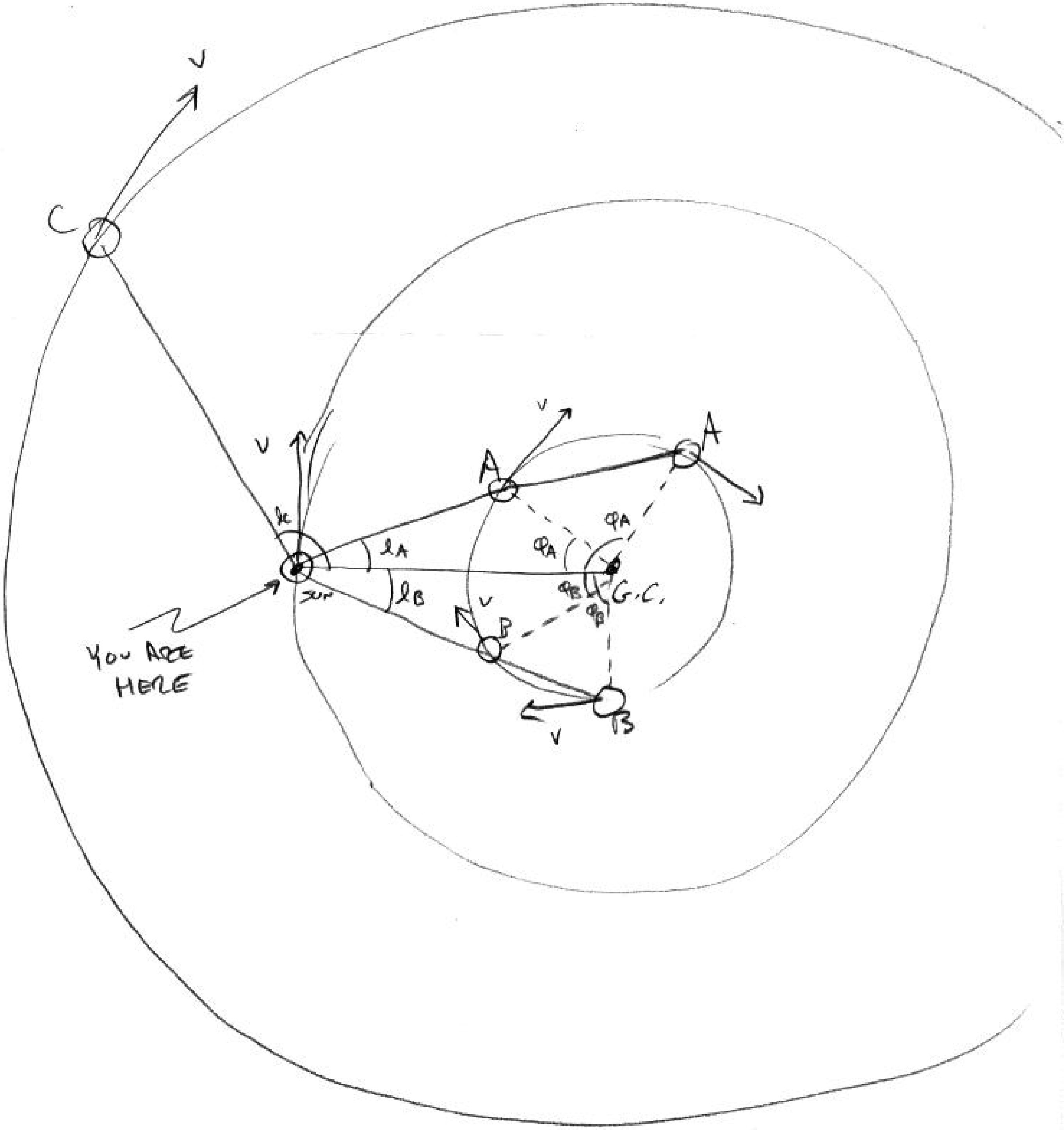
$$\boxed{d = R_o \cos l \pm \sqrt{R^2 - R_o^2 \sin^2 l}}$$

(c)

BLOB	R (kpc)	d (kpc)	φ (FOR PLOTTING)
A	4.0	4.96 or 9.68	30° or 102°
B	4.0	5.37 or 8.93	-37° or -90°
C	12.0	6.46	30°

$R > 0$, ∴ ONLY ONE ANSWER FOR C

14 d


C
v
v
A
v
A
l_C
l_A
l_B
φ_A
φ_A
φ_B
φ_B
G.C.
SUN
v
B
B
v
YOU ARE HERE