1 (a) SURFACE BRIGHTNESS DOESN'T DEPEND ON DISTANCE*; SO PRETEND THE GALAXY IS AT $d = 10pc$. THAT SIMPLIFIES THINGS, BUT THE RESULT SHOULD STILL BE GENERALLY APPLICABLE

CALL "F" THE FLUX FROM ONE pc^2 OF THE GALAXY

IF IT IS @ 10pc

LUMINOSITY FROM ONE $\square''$

$$\frac{F}{F_{\odot 10pc}} = \frac{L}{L_\odot}$$

SINCE $d = 10pc$ FOR BOTH

FLUX OF SUN @ 10pc

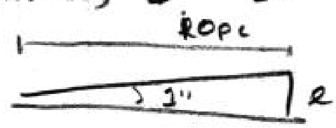
$$m - M_\odot = -2.5 \log \frac{F}{F_{\odot 10pc}} = -2.5 \log \frac{L}{L_\odot}$$

mag of sun @ 10pc

$$20 - 4.8 = -2.5 \log \frac{L}{L_\odot}$$

$$L = L_\odot \, 10^{(20-4.8)/-2.5} = 8.3 \times 10^{-7} L_\odot$$

AT THIS DISTANCE, 1" IS:



$$\frac{\ell}{10pc} = (1'')\left(\frac{1\,rad}{206265''}\right)$$

$$= \frac{10}{206265} pc = 4.8 \times 10^{-5} pc$$

THUS, $$\frac{L}{A} = \frac{8.3 \times 10^{-7} L_\odot}{(4.8 \times 10^{-5} pc)^2} = \boxed{350 \frac{L_\odot}{pc^2}}$$

ANOTHER WAY TO DO THIS: KEEP THE DISTANCE ARBITRARY

$$F = \frac{L}{4\pi d^2}$$

WHATEVER IT IS

$$F_{\odot 10pc} = \frac{L_\odot}{4\pi (10pc)^2}$$

⇒

1 (a) cont'd

$$m - M_\odot = -2.5 \log \frac{F}{F_{\odot, 10pc}} = -2.5 \log \frac{L\,(10pc)^2}{L_\odot d^2}$$

$$20 - 4.8 = -2.5 \log \frac{L\,(10pc)^2}{L_\odot\, d^2}$$

$$L = L_\odot \left(\frac{d}{10pc}\right)^2 10^{(20-4.8)/-2.5} = 8.32\times10^{-9}\, L_\odot \left(\frac{d}{1pc}\right)^2$$

$$\frac{\ell}{1pc} = \frac{1}{206265} \text{ rad}$$

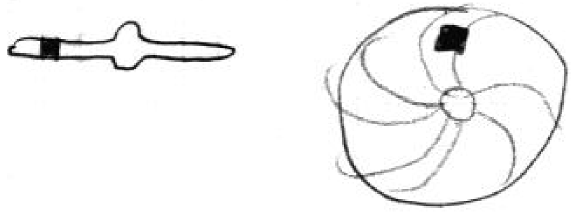
$$\ell = \frac{d}{206265}$$

d DIVIDES OUT!

$$\frac{L}{\ell^2} = \frac{8.32\times10^{-9}\, L_\odot\, \cancel{d^2}\, (206265)^2}{\cancel{d^2}\, (1pc)^2}$$

$$\boxed{\frac{L}{\ell^2} = 350 \frac{L_\odot}{pc^2}}$$

(b)



(c) $M/L \sim 0.5$ FOR GALACTIC DISK

THUS WE HAVE $\boxed{\sim 700\, M_\odot \text{ IN } 1\, pc^2}$

(d) FIG 2.3: # STARS IN $1000 pc^3 \approx 1.5 + 3.5 + 5 + 7 + 10 + 7.5 + 7.2.5 + 2 + 7.5 + 2 + 1.5 + 0.5 \approx 53$

MASS OF STAR IN $1000 pc^3 \approx (3)(1.5) + 3(3) + 1 + 4(1.5) \approx 20$

$\frac{\#\text{STARS}}{M_\odot} \approx 2.7$ THUS $\sim$ $\boxed{\text{2000 STARS}}$

2

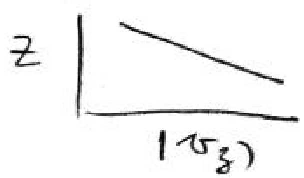
- CONSIDER THE THIN DISK
  - OLDER STARS HAVE <u>LOWER</u> METALLICITY GENERALLY

    (IF STARS ARE OLD, THEY FORMED EARLY IN THE GALAXY'S HISTORY BEFORE GAS CLOUDS WERE FULLY ENRICHED WITH HEAVY ELEMENTS)

  - OLDER STARS TEND TO HAVE <u>HIGHER</u> $|v_z|$

    (START NEAR GALACTIC PLANE MIDPOINT WHERE THE GAS IS. AS TIME GOES BY, THERE MORE LIKELY TO BE "SCATTERED" TO HIGHER z POSITIONS, AND THUS HIGHER $|v_z|$ TO GET THERE.

  - THUS, YOU'D EXPECT METALLICITY TO GO DOWN AS $|v_z|$ GOES UP



- ADD TO THIS THE HALO & THICK DISK. THESE ARE LOW-METALLICITY COMPONENTS SPREAD OVER LARGER z THAN THE THIN DISK (esp. THE HALO!), AND THUS WILL TEND TO ENHANCE THIS SAME TREND

- EXPECT LOTS OF NOISE IN THE TREND!

3 (a)

$$I(R) = I(0)\, e^{-R/h_R}$$

CENTRAL VALUE

$$\frac{I(0)}{2} = I(0)\, e^{-R/h_R}$$

$$\ln \frac{1}{2} = -\frac{R}{h_R} = -0.69$$

$$\boxed{R = 0.69\, h_R}$$

$$\boxed{R = 3.5 \text{ kpc}}$$

⇒

13(b) FOR A DISK:

$$\frac{\text{LIGHT @ } R < 0.69\,h_r}{\text{ALL LIGHT}} = \frac{\int_0^{2\pi}\int_0^{0.69h_R} \cancel{I(0)}\, e^{-R/h_R}\, R\,d\varphi\,dR}{\int_0^{2\pi}\int_0^{\infty} \cancel{I(0)}\, e^{-R/h_r}\, R\,d\varphi\,dR}$$

THIS INTEGRAL: $\int R e^{-R/h_R} dR$

INTEGRATE BY PARTS: $\int u\,dv = uv - \int v\,du$

$u = R \quad du = dR$
$v = -h_R e^{-R/h_R} \quad dv = e^{-R/h_R} dR$

$$= -h_R R e^{-R/h_R} + h_R \int e^{-R/h_R} dR$$

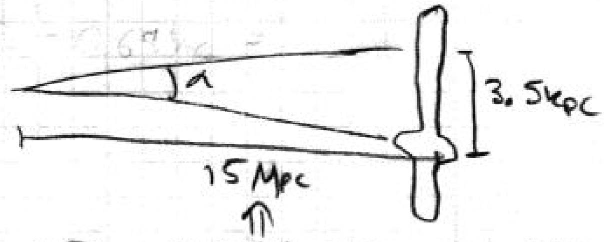
$$= -h_R R e^{-R/h_R} - h_R^2 e^{-R/h_R} dR$$

$$= \frac{\cancel{2\pi h_R}\left[R e^{-R/h_R} - h_R e^{-R/h_R}\right]_{R=0}^{R=0.69h_R}}{\cancel{2\pi} h_R \left[R e^{-R/h_r} - h_R e^{-R/h_r}\right]_{R=0}^{R=\infty}}$$

$$= \left[\frac{0.69h_R\left(\frac{1}{2}\right) - h_R\left(\frac{1}{2}\right) + h_R}{h_R}\right]$$

$$= 0.35 - \frac{1}{2} + 1 = \boxed{85\%}$$

3 d



15 kpc IS **WAY** TOO CLOSE FOR A GALAXY; I **MEANT** 15 Mpc

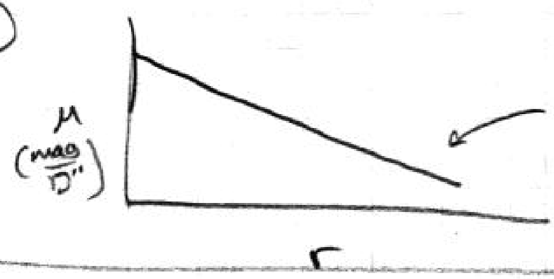
$$\alpha = \left(\frac{3.5 \times 10^3 \text{ pc}}{15 \times 10^6 \text{ pc}} \text{ rad}\right)\left(\frac{206265''}{\text{rad}}\right) = \boxed{48''}$$

e

$$\frac{I(\tfrac{1}{2} I_0)}{I_0} = \frac{1}{2}$$

$$m(\tfrac{1}{2} I_0) - m_0 = -2.5 \log \frac{1}{2}$$
$$= 0.75$$
$$m(\tfrac{1}{2} I_0) = m_0 + 0.75 = \boxed{15.75 \ \frac{\text{mag}}{\square''}}$$

f



LINE 'CAUSE mag IS A LOG, I IS AN EXPONENTIAL

4

RATE OF STARS BEING MADE = $C(M)\, e^{-t/t_*}$ ← MASS-DEPENDENT CONSTANT

TOTAL # $2 M_\odot$ STARS MADE

$$= \int_0^{\tau_{gal}} C(2M_\odot)\, e^{-t/t_*}\, dt$$

$$= C(2M_\odot)\left[-t_* e^{-t/t_*}\right]_0^{\tau_{gal}}$$

$$= C(2M_\odot)\, t_* \left(1 - e^{-\tau_{gal}/t_*}\right)$$

ONLY STARS IN THE LAST 1.1 Gyr (TAB. 1.1) ARE STILL AROUND...

$$\#_{MS} = \int_{\tau_{gal} - 1.1\,\text{Gyr}}^{\tau_{gal}} C(2M_\odot)\, e^{-t/t_*}\, dt$$

⇒

$$\#_{MS} = C(2M_\odot)\left[-t_* e^{-t/t_*}\right]_{\tau_{gal}-1.1\,Gyr}^{\tau_{gal}}$$

$$= C(2M_\odot)t_*\left[e^{-(\tau_{gal}-1.1Gyr)/t_*} - e^{-\tau_{gal}/t_*}\right]$$

$$\frac{\#_{MS}}{\#_{MADE}} = \frac{e^{-(\tau_{gal}-1.1Gyr)/t_*} - e^{-\tau_{gal}/t_*}}{1-e^{-\tau_{gal}/t_*}}$$

FOR $\tau_{gal} = 10\,Gyr$, $t_* = 3\,Gyr$

$$= \frac{e^{-(8.9\,Gyr)/3\,Gyr} - e^{-10Gyr/3Gyr}}{1-e^{-10Gyr/3Gyr}}$$

$$= \boxed{0.016}$$ (2% TO ONE SIG FIG)

FOR $3M_\odot$ STARS, REPLACE 1.1 Gyr WITH 0.35 Gyr TO GET

$\boxed{0.005 \text{ OF } 3M_\odot \text{ STARS STILL AROUND}}$

⇒ EQUATION 2.4 PREDICTS $\Psi(M_V)$ (INITIAL LUMINOSITY FUNCTION OR ILF) GIVEN $\Phi(M_V)$ UNDER THE ASSUMPTION OF CONSTANT RATE.

IF THE RATE IS CHANGING, THEN THE NUMBER STILL AROUND TODAY IS THE FORMATION RATE INTEGRATED OVER THE LAST τ_{MS}. ~~F~~FOR LONGER-LIVED STARS, YOU ARE INTEGRATING OVER TIMES FURTHER BACK WHEN THE RATE WAS HIGHER, WHEREAS FOR SHORTER-LIVED STARS, YOU'RE ONLY INTEGRATING OVER THE RECENT, LOWER RATE.

THIS, AS LIFETIME ↓, Φ WILL BE <u>LOWER</u> COMPARED TO Ψ. THE SHORTER THE LIFETIME, THE MORE THE NAIVE CALCULATION UNDERESTIMATES Ψ.

FOR $\tau_{MS} > \tau_{gal}$, Ψ ISN'T AFFECTED.