Astronomy 253 — Solutions to Group Problems 1

Friday, January 23

1. (a) Looking at the upper plot, the star has the highest F_λ at about 4000 Å, so that's where you'd want to put your filter.

   (b) Looking at the figure in the book, the B filter is at about 4500 Å, while the R filter is at about 6500 Å. They are pretty similar in width; the R filter may be a bit wider, but not by much. (A wider filter will admit more energy than a narrower one at the same wavelength, because it's letting photons of more colors, and therefore more photons, through.)

   Star 1 has a greater F_λ at 4500 Å, so it will have a greater flux through the B filter than through the R filter. Star 2, on the other hand, has a greater F_λ at 6500 Å, so it will have a greater flux through the R filter.

   (c) We have established that $F_B/F_R > 1$ for Star 1 and $F_B/F_R < 1$ for Star 2. This means that Star 1 has a greater F_B/F_R. B-R is *higher* when the star is *redder*, which is the case for Star 2, so Star 2 should have a larger value of B-R. Mathematically,

$$\begin{aligned} B - R &= -2.5 \log \left(\frac{F_B/F_{B0}}{F_R/F_{R0}} \right) \\ &= -2.5 \log \left(\left[\frac{F_B}{F_R} \right] \left[\frac{F_{R0}}{F_{B0}} \right] \right) \end{aligned}$$

   A larger F_B/F_R means a smaller B-R, due to the negative sign on the 2.5.

   (d) Something redder! Maybe yellowish, orangish, or reddish.

2. (a) Clearly there are a larger number with $L < L_*$. Even if there are no galaxies with absolute magnitudes dimmer than -16, the total number you can add up between -16 and -20.5 (which is about the magnitude that corresponds to L_*) is much larger than the number you'd add up for brighter galaxies, where the number drops off sharply with greater magnitudes.

   (b) The caption in the book indicates that the dashed line is the total amount of luminosity coming from galaxies at each magnitude (really, within each range of $\Delta M = 1$ magnitude). The total light coming from galaxies dimmer than L_* would be the sum of all the points along this line up to L_*, that is, the integral of this line from the left side of the plot to L_*. Similarly for $L > L_*$. Graphically, you can do integrals by eyeballing the area underneath the curve.

   The logarithmic axis on the vertical axis makes this a little bit complicated to do graphically. (Note that although the horizontal axis *looks* logarithmic, remember that what is plotted is the number of galaxies per range of *magnitude*; magnitudes are on the top of the plot, and represent a linear plot). Just comparing the area under the curve to the left and right of L_* suggests that the luminosities should be similar. The logarithmic vertical axis, however, gives a small boost to the right side, where the peak of the curve gets, so in fact this plot would indicate that a bit more of the light in the Universe comes from $L > L_*$ galaxies (although the two are comperable). Note that very little of the light in the Universe comes from $L < 0.1L_*$ galaxies!

(c) M31 and the Milky ay are both a bit above L_*, M33 is a bit below L_* (close to the "5" on the bottom axis), the LMC is just above $0.1L_*$, and the rest are below $0.1L_*$. A distant observer looking at the Local Group would probably see it as a collection of three spiral galaxies, two big and bright ones (us and M31), and one smaller one (M33); many of the rest of the galaxies would be harder to pick out. Indeed, if we look at groups of galaxies in the Universe, we typically see a handful of bright galaxies; there are almost certainly more dwarf galaxies in those groups than we've generally found.

(d) The Milky Way is a galaxy with $L\ L_*$, which puts it in a select group of the brightest galxaies. It's not a big huge cD galaxy, but it's pretty studly as galaxies go.

And, indeed, although the Sun as a G-star is pretty wimpy compared to bright hot blue stars, it's pretty studly compared to the M-dwarfs that make up the bulk of the stellar population.

Does this mean curtains for the Cosmological Principle? Since we're in a big galaxy, as galaxies go, and in an above-average mass star, as stars go, does this mean that we're in a special place in the Universe? No. There are lots and lots of other galaxies very much like the Milky Way out there. Indeed, whether or not we should be surprised to bein the Milky Way requires deeper thought. We know that a good fraction of the light of the Universe comes from the brightest galaxies; where is most of the mass of the Universe? If the "initial mass function" (I.e. how many stars form at different masses) is the same everywhere, then there are about as many stars in giant galaxies as in dwarf galaxies, so it's no real surprise we're in a giant galaxy.

Similarly, there are lots and lots of stars like the Sun. There are reasons why it may be easier for live to evolve around a star like the Sun, as compared to an M-dwarf. (A larger range of distances from the star can support liquid water.)

3. When Hydrogen is converted to Helium, about 0.7% of its mass is converted to energy. The luminosity of the Sun is $L_\odot = 3.86 \times 10^{26}$ W (see p. 2), so:

$$\begin{aligned} 0.007\, m_{\mathrm{conv}}\, c^2 &= 3.86 \times 10^{26}\ \mathrm{J} \\ \boxed{m_{\mathrm{conv}} = 6 \times 10^{11}\ \mathrm{kg}} \end{aligned}$$

each second. The Sun has a total mass of 2×10^{30} kg, so it could last:

$$\frac{2 \times 10^{30}\,\mathrm{kg}}{6 \times 10^{11}\,\mathrm{kg/s}} = 3 \times 10^{18}\,\mathrm{s} = 10^{11}\,\mathrm{yr}$$

An O5-star has a Luminosity that is 240,000 times greater, which means that it is using up mass at 240,000 times the rate, or 1.5×10^{16} kg/s. The O-star has 40 times the mass but is using its fuel at 240,000 times the rate, so it will only last 40/240,000 times as long, or about 2×10^7 year (if it were to use all the fuel available; in fact, it lasts only something like a tenth as long).

4. If two stars of the same luminosity form a close binary pair, they will have the same flux as observed from Earth. Thus, the total flux is:

$$F_{\text{tot}} \; = \; F_1 \; + \; F_2 \; = \; 2\,F_1$$

   Comparing the magnitude of the two together to one of the stars is a simple matter of taking the ratio of the fluxes and doing the right logarithmic magic with it:

$$\begin{aligned} m_{\text{tot}} \; - \; m_1 \; &= \; -2.5 \log\left(\frac{F_{\text{tot}}}{F_1}\right) \\ &= \; -2.5 \log\left(\frac{2\,F_1}{F_1}\right) \\ &= \; -2.5 \log(2) \; = \; -2.5\,(0.30) \\ &\boxed{m_{\text{tot}} \; - \; m_1 \; = \; -0.75} \end{aligned}$$

   Note that the negative difference means that m_tot has a smaller value; smaller magnitude means *brighter*.

5. The main-sequence K2 star must be much closer. Those stars are intrinsically much less luminous than O5 stars, so to appear the same magnitude (and thus the same flux or brightness) from Earth, the K star must be a lot closer.

   We can relate flux to distance with:

$$F_{\text{O5}} \; = \; \frac{L_{\text{O5}}}{4\,\pi\,{d_{\text{O5}}}^2} \qquad F_{\text{K2}} \; = \; \frac{L_{\text{K2}}}{4\,\pi\,{d_{\text{K2}}}^2}$$

   Since the two have the same magnitude, we know they have the same flux, so:

$$\begin{aligned} \frac{F_{\text{O5}}}{F_{\text{K2}}} \; &= \; 1 \; = \; \frac{L_{\text{O5}}/4\,\pi\,{d_{\text{O5}}}^2}{L_{\text{K2}}/4\,\pi\,{d_{\text{K2}}}^2} \\ 1 \; &= \; \left(\frac{L_{\text{O5}}}{L_{\text{K2}}}\right)\left(\frac{d_{\text{K2}}}{d_{\text{O5}}}\right)^2 \\ \frac{d_{\text{O5}}}{d_{\text{K2}}} \; &= \; \sqrt{\frac{L_{\text{O5}}}{L_{\text{K2}}}} \end{aligned}$$

   You can find luminosities for these types of stars in Table 1.1 on p. 10; putting those values in gives:

$$\frac{d_{\text{O5}}}{d_{\text{K2}}} \; = \; 1,000$$

The O5 star is 1,000 times farther away than the K2 star if they appear to be the same magnitude.

6. (a) G stars must be much more common. Main-sequence A stars are intrinsically much more luminous than main-sequence G stars, so an A-star at magnitude 18 will be much farther away than a G-star at magnitude 18 (see previous problem). You're sampling a much greater volume of A-stars, but only counted as many of those as you did G-stars, so the number density of G-stars is a lot greater.

   (b) The calculation here is already done in the previous problem! The only difference is the luminosities; plugging in the right values for A and G-stars gives:

   $$\frac{d_A}{d_G} = 5$$

   This means that you were sampling a volume that was $5^3 = 125$ times bigger when looking for A-stars. So, in that whole volume, there should be 125 times as many main-sequence G stars as main sequence A stars.

7. (a) Very roughly, for collisional ionization, you need:

   $$kT \simeq 13.6\,\text{eV}$$

   Putting in the right value for Boltzmann's constant k, this gives us a temperature of about 1.6×10^5 K.

   (b) Similarly, for an energy gap of 10.2 eV, you get 1.2×10^5 K.

   (c) A giant A-star will be much larger in radius (and therefore volume) than a main-sequence A-star, but probably not that different in mass. Thus, you'd expect the giant to have a *lower* density at its surface than the dwarf. (This really isn't as obvious as it may seem, since we're talking about the *atmosphere* of the star—that's where the absorption lines happen. However, the lower surface gravity of the giant star would lead you to expect it to have a more spread out, lower-density atmosphere.)

   If the density is high enough, you'd expect a pure statistical equilibrium, with the ratio of Hydrogen in the n=2 state to Hydrogen in the n=1 state equal to $e^{-10.2\,eV/kT}$. However, if the density is low enough that the mean time between collisions is comperable to or less than the amount of time it takes the $n = 2$ state to radiatively decay, then the n=2 state will be underpopulated. As the frequency of collisions goes up, then the rate at which you can repopulate that n=2 state goes up. Where the density is higher, you'd expect more frequent collisions. Thus, you'd expect that the dwarf A-star, which has a higher density at it's surface, would have a greater fraction of its Hydrogen in the n=2 state.

   (d) The balmer Hydrogen absorption lines all represent the case where Hydrogen in the n=2 state absorbs a photon and jumps to a higher state. Thus, the strength of the line (not the width, but the total amount of "missing light" in the line) will go up as there is more n=2 Hydrogen available. You'd expect stronger Balmer absorption lines in a dwarf star than a giant star, following this reasoning.