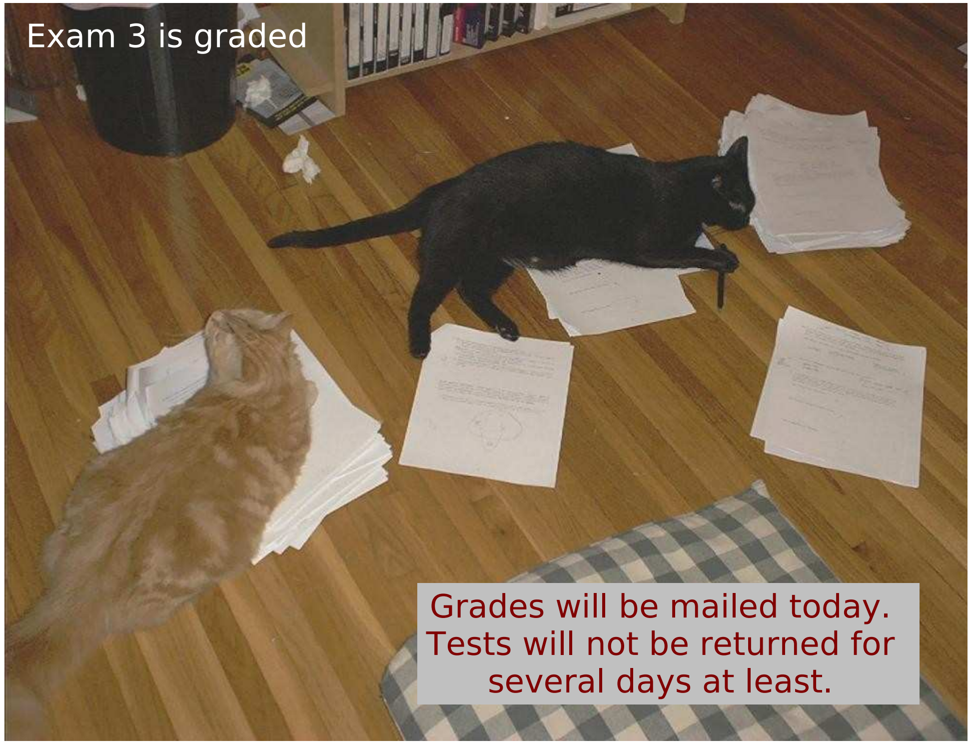


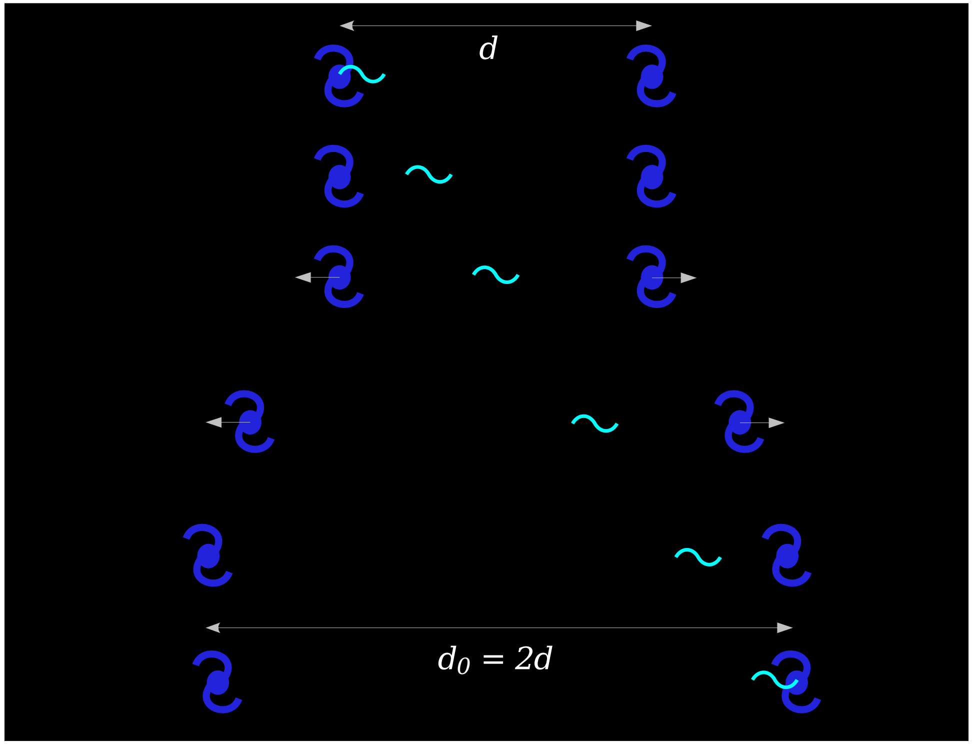


The expanding Universe equation : $z = \dfrac{d}{c\ t_H}$
(“Hubble's Law”)

**The Meaning of the Variables**

$d$ = distance the light traveled (= distance to the galaxy for $z \ll 1$)

$c$ = speed of light

$t_H$ = Hubble time

$z$ = cosmological redshift

“Size” of the Universe, o something proportional to it (avg. dist. bet. galaxies, dist. to a given distant galaxy)

$$z = \frac{\text{Change in Size}}{\text{Size}}$$

$$1+z = \frac{\text{Size Now}}{\text{Size When Light Emitted}}$$

We observe a quasar. The light we observe was emitted when the Universe was 1/3 its current size. What is the redshift $z$ of the quasar?

A $z$=0.33

B $z$=0.5

C $z$=1

D $z$=2

E $z$=3

$$1+z = \frac{\text{Size Now}}{\text{Size Then}}$$

$$1+z = \frac{3}{1} = 3$$

$$z = 2$$

***The top rung of the cosmic distance ladder...***

## ***Redshift***

The Universe has always been expanding....

...thus, greater distance = larger lookback time = more total expansion since emission = bigger $z$

$$z = \frac{d}{c\,t_H} \qquad\qquad d = c\,t_H\,z$$

Redshift is almost always easier (though expensive in telescope time) to measure than distance... but using it as a stand-in for distance requires knowing $t_H$ (and, actually, a few other things).

*Warning : this equation should only be used for $z \ll 1$. For larger $z$, the interpretation of $d$ gets complicated.*

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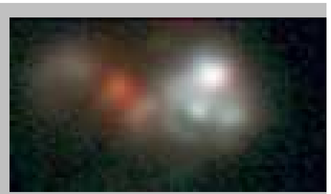

**Example** : Prof. Knop's favorite galaxy is VV114. VV114 has a redshift $z$=0.020. How far away is VV114?

$$d = c\,t_H\,z$$

$$d = \left(1\ \frac{\text{light-year}}{\text{year}}\right)(13.8\times10^9\ \text{years})\,(0.020)$$

$$d = 2.76\times10^8\ \text{light-years} = 278\ \text{Mlyr}$$

$$d = (2.76\times10^8\ \text{lyr})\left(\frac{1\ \text{pc}}{3.26\ \text{lyr}}\right) = 8.47\times10^7\ \text{pc} = 84.7\ \text{Mpc}$$

---

What is Prof. Knop's Favorite Galaxy?

- ~~A The Milky Way~~ *Annoying people live there!*
- ~~B The Andromeda Galaxy~~ *It's so close!!!*
- C VV114 *So pretty!!! And, anyway, it's always C.*
- ~~D M87~~
- ~~E M81~~

*People get them mixed up!!! (Although I did win $0.05 on a bet with a seminar speaker on which was which...)*

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The Quasar 3C273 is the closest quasar to the Milky Way. (There are closer objects that are similar to quasars, but not as luminous.) It has a measured redshift of z=0.158. How far away is it in Mpc? (For a sense of scale, recall that the Andromeda Galaxy is about 0.8 Mpc away, and the Virgo Cluster is about 20 Mpc away.)

Hubble's Law : $z = \dfrac{d}{c\ t_H}$ *(Note: works for $z \ll 1$)*

$$d = z\,c\,t_H$$

$$d = 0.158\ \left(3\times10^8\,\frac{\text{m}}{\text{s}}\right)\ (13.8\times10^9\,\text{yrs})$$

$$d = 0.158\ \left(3\times10^8\,\frac{\text{m}}{\text{s}}\right)\ (13.8\times10^9\,\text{yrs})\left(\frac{3.16\times10^7\,\text{s}}{\text{yr}}\right)\left(\frac{1\,\text{Mpc}}{3.086\times10^{22}\,\text{m}}\right)$$

$d$ = 670 Mpc

## An easier way to do it (fewer unit conversions):

Hubble's Law : $z = \frac{d}{c\ t_H}$ *(Note: works for $z \ll 1$)*

$$d = z\,c\,t_H$$

$$d = 0.158\ \left(1\frac{\text{lyr}}{\text{yr}}\right)\ (13.8\times10^9\,\text{yrs})$$

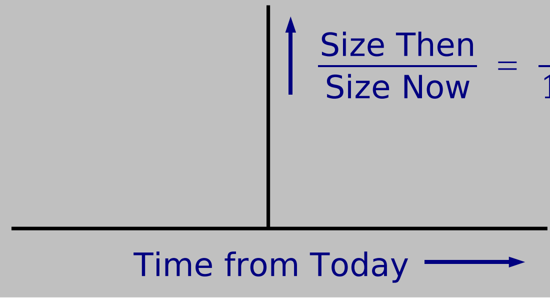
$$d = 2.2\,\text{billion light years}$$

$$d = (2.2\times10^9\,\text{lyr})\left(\frac{1\,\text{Mpc}}{3.26\times10^6\,\text{lyr}}\right) = 670\ \text{Mpc}$$

## Measuring the Expansion of the Universe:

1. Measure the distance to figure out the lookback time (farther away = light was traveling longer to reach us).
2. Measure the redshift to find out how much the Universe expanded during that time.

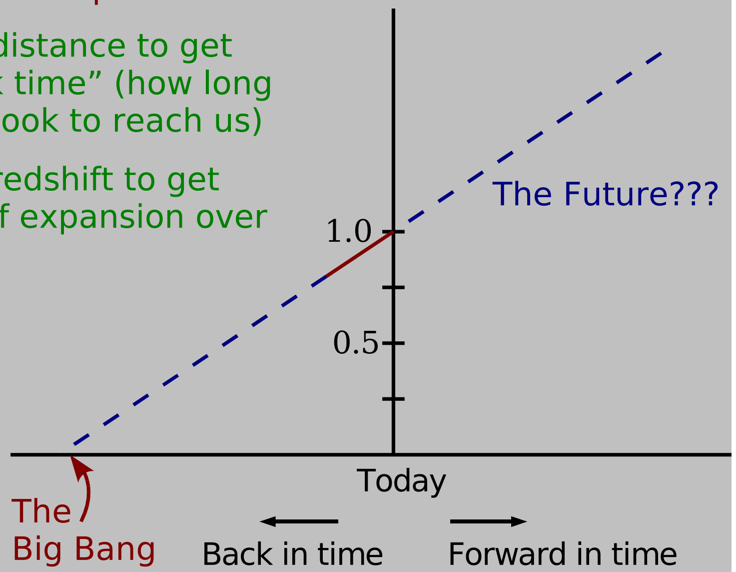
$$\frac{\text{Size Then}}{\text{Size Now}} = \frac{1}{1+z}$$




### Measuring the Expansion

$$\frac{\text{Size}}{\text{Size Today}} = \frac{1}{1+z}$$

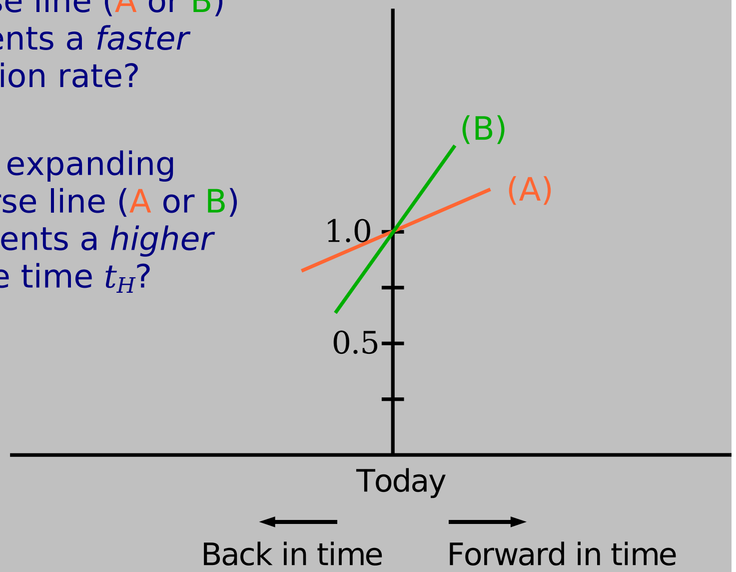
- Measure distance to get "lookback time" (how long the light took to reach us)
- Measure redshift to get amount of expansion over that time.

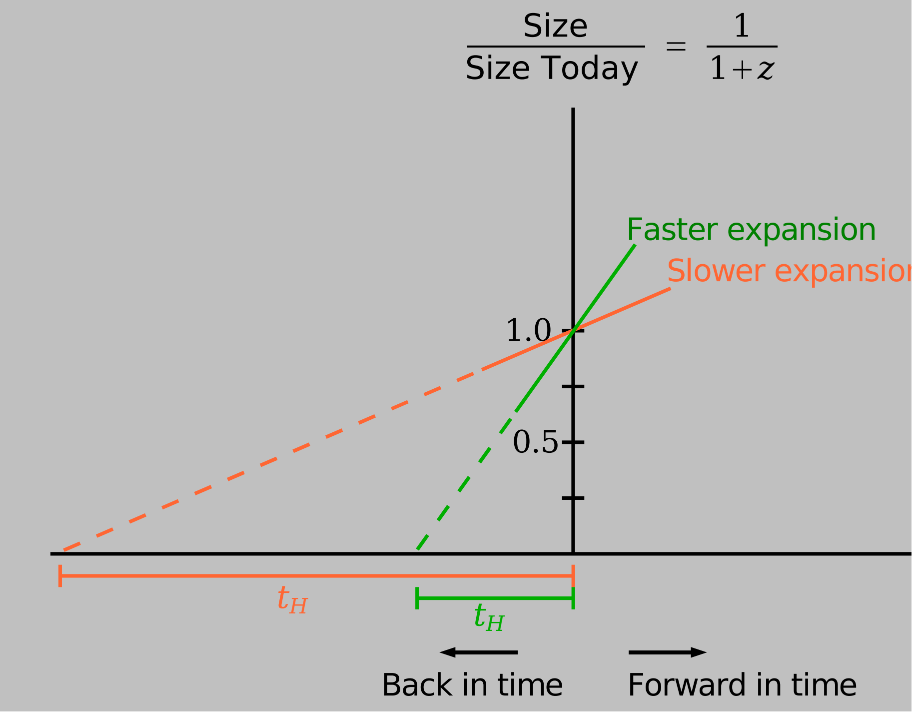



Q1 Which expanding Universe line (A or B) represents a *faster* expansion rate?

Q2 Which expanding Universe line (A or B) represents a *higher* Hubble time $t_H$?

$$\frac{\text{Size}}{\text{Size Today}} = \frac{1}{1+z}$$






## The meaning of the Hubble Time $t_H$

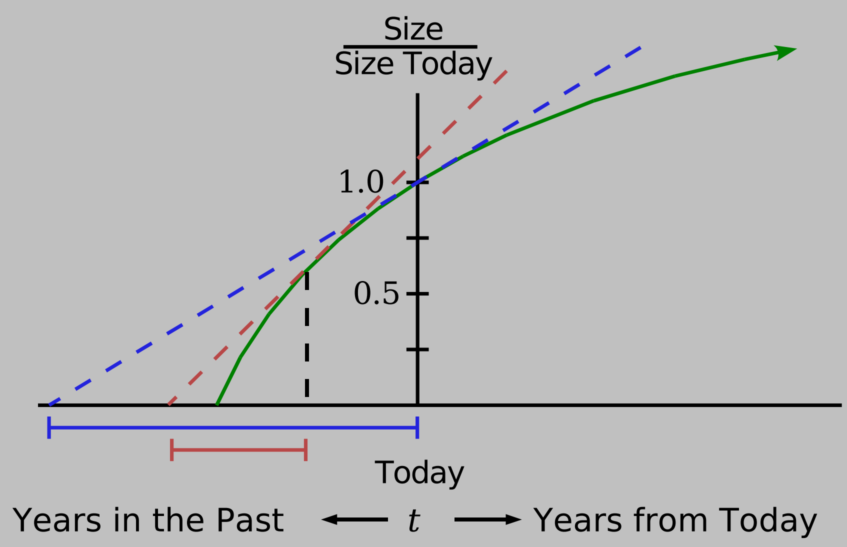
$$z = \frac{d}{c\ t_H}$$

$t_H$ gives you the expansion rate of the Universe:
Higher $t_H$ = slower expansion

$t_H$ is the “doubling time” – how long it will take the size of the Universe to double from right now iif the expansion rate remains constant.

$t_H$ is the age of the Universe *if* the expansion rate has always been constant and equal to the current expansion rate.

If the expansion is slowing down:
*expansion rate* was higher in the past than it is today




If the expansion is slowing down:
The age of the Universe is less than $t_H$