$$E = m c^2$$

Concept Inventory

- The scale of the solar system, nearby stars, the milky way, the Universe.
- Radioactive decay as a stochastic process (random, but with precise probabilities)
- Radiometric dating and the age of the Solar System
- amount = rate × time
- $E = mc^2$, converting mass to energy
- Efficiency of using fuel in energy generation
- The Sun is powered by nuclear fusion

A Carbon-14 atom has a half-life of 5,700 years. If you have a Carbon-14 atom, approximately what is the probability that it remains undecayed a second later?

A Each individual atom has a 50% chance of decaying each second.

B There is no way of telling when the Carbon-14 atom will decay, but the chances of it NOT decaying is about 70% (according to dice/unstable isotope chart from class notes).

C .5/(1.79874478 × 10^{11} seconds)

D It will have a roughly a 99% chance that it remains undecayed.

E The probability that a Carbon-14 atom remains undecayed a second later would be about 100%.

You have a Carbon-14 atom. You watch it for 5,700 years. At the end of that time, you see that it is still there.

What would you then estimate is the probability that the Carbon-14 atom is still around after another 5,700 years?

A The probability is not very high. Just because it did not decay exactly when its half life arrived does not mean that it is some mutant atom and that it is going to survive another half life. However, it is simply probability, so there is a small chance that it could survive for another 5700 years without decaying, but the chance of that happening is not very good.

B The probability is still 50%.

C The probability would be close to 75% because the first half-life that passes takes 50% of the probability, and the next half life starts over so it would be taking half of the 50% that is left, or 25%. So when you add the 2 half-lives together, you get a total of 75% probability that it has decayed, so there is a 25% chance it will still be around.

The probability is still 50%. In an instance like this, it is easy to fall prey to the 'gambler's fallacy;' however, the decay of a Carbon-14 atom is a semi random event. If, at the end of 5700 years the atom has not yet decayed, there is still only a 50% chance it will decay in the next 5700 years.

GUILDENSTERN: *Four: a spectacular vindication of the principle that each individual coin spun individually* [he spins one] *is as likely to come down heads as tails and therefore should cause no surprise each individual time it does.* [It does. He tosses it to ROS.]

Your mass at the beginning of the day
+ all mass taken in (eating, breathing...)
– all mass given out (breathing, bathroom, sweat...)

compared to

Your mass at the end of the day.

A Your mass at the end of the day is *higher* by a tiny fraction of a pound.

B Your mass at the end of the day is *higher* by a few hundredths of a pound (leading to weight gain of a few to several pounds a year).

C They are the same.

D Your mass at the end of the day is *lower* by a few hundredths of a pound (leading to weight loss of a few to several pounds a year).

E Your mass at the end of the day is *lower* by a tiny fraction of a pound.

$M_\odot = 2\times10^{30}$ kg — Sun's Mass

$L_\odot = 3.8\times10^{26}$ W — Sun's rate of energy generation (1W = 1 J/s)

$t_\odot = 4.6\times10^{9}$ yr $= 1.45\times10^{17}$ s — Age of the Solar System

eff for chemical reactions : $\sim1\times10^{-10}$

What is the total amount of energy the Sun could generate through chemical reactions if it used *all* of its mass as fuel?

$$eff = \frac{E_{\text{produced}}}{m_{\text{fuel}}\, c^2} \qquad E_{\text{produced}} = (eff)\,(m_{\text{fuel}}\, c^2)$$

$$E_{\text{produced}} = (eff)\,(m_{\text{fuel}}\, c^2)$$

$$E_{\text{produced}} = \left(10^{-10}\right)\left(2.0\times10^{30}\text{ kg}\right)\left(3.0\times10^{8}\,\frac{\text{m}}{\text{s}}\right)^2$$

$$E_{\text{produced}} = 2\times10^{37}\text{ J}$$

This is how much energy the Sun could produce through chemical reactions using up ***all*** of its mass as fuel.

It is using energy at a rate of 3.8×10^{26} J/s. How long would it last?

$$t = \frac{\text{amount}}{\text{rate}} = \frac{2\times10^{37}\text{ J}}{3.8\times10^{26}\text{ J s}^{-1}} = 5\times10^{10}\text{ s}\left(\frac{1\text{ year}}{3.16\times10^{7}\text{ s}}\right)$$

$$= 2{,}000\text{ years}$$

To have been shining at its current luminosity for a few billion years, the sun needs an energy generation process that has an efficiency of about 10^{-3} (or even a bit better).

Chemical reactions (normal "burning") have efficiencies of about 10^{-10}.

If the Sun were "on fire" (generating its energy via chemical reactions), how old could it be?

- A Billions (10^9) of years
- B Millions (10^6) of years
- C Thousands (10^3) of years
- D Just a few years old
- E It would burn out in a few days

Lower efficiency (*eff*) = using up fuel faster to generate the same amount of energy.

If two processes with different efficiencies are generating energy at the same rate, and have the same total amount of fuel, which one is going to have the *shorter* lifetime?

- A The one with higher *eff*
- B The one with lower *eff*
- C If the *amount* of fuel is the same, and the *rate* of energy generation is the same, they will both have the same *time*, or lifetime.

(amount of *fuel used*) = (rate of *fuel use*) × (time)

To have been shining at its current luminosity for a few billion years, the sun needs an energy generation process that has an efficiency of about 10^{-3} (or even a bit better).

Chemical reactions (normal "burning") have efficiencies of about 10^{-10}.

If the Sun were "on fire" (generating its energy via chemical reactions), how old could it be?

Chemical reactions have an eff that is only

$$\frac{10^{-10}}{10^{-3}} = 10^{-7}$$ what it needs to be.

Thus, they use fuel 10^7 times faster than the real Sun, so using them the Sun would live 10^{-7} as long, or a few *hundred* years.

Nuclear Fusion

$4H \rightarrow He + 2e^+ + 2\nu$ + energy

H = Hydrogen nucleus = proton (p)
He = Helium nucleus = alpha particle (α)

Really: $4p \rightarrow \alpha + 2e^+ + 2\nu$ + energy

m_p = 1.00728 amu
m_α = 4.00151 amu
m_e = 0.00055 amu
m_ν < 0.00000 amu

1 amu = 1.66×10^{-27} kg

Mass of 4p = 6.691×10^{-27} kg

Mass of $\alpha + 2e^{+} = 6.646\times10^{-27}$ kg

Difference in mass : 4.5×10^{-29} kg

Energy released : $(4.5\times10^{-29}\text{ kg})(3\times10^{8}\text{ m/s})^2$
$= 4.0\times10^{-12}$ J

$$eff = \frac{4.5\times10^{-29}\text{ kg}}{6.691\times10^{-27}\text{ kg}}$$

$$= 0.007 = 0.7\%$$

DING!

How much mass does the Sun lose during its lifetime?

0.7% = 0.007 of the mass *used in Hydrogen Fusion* is converted converted away to energy.

During the main part of the Sun's lifetime, it will use approximately 10% = 0.1 of its mass as fuel for the Hydrogen fusion process.

Mass converted to energy
$= 0.007\ (\ (0.1)\ (2\times10^{30}\text{ kg})\ )$
$= 1.4\times10^{27}$ kg

That sounds like a lot, but is <0.1% of the Sun's mass!